Abstract

Timber frame structures often rely on pegged mortise and tenon connections to secure members together. Wood pegs, while large in diameter, are considerably more flexible than a steel dowel of the same size. Engineering design of these pegged tension connections is not addressed in the National Design Specification for Wood Construction (NDS) [1]. In an effort to standardize the design procedure used for timber frame structures, the TFEC has developed the Standard for Design of Timber Frame Structures TFEC 1-10 [2], which includes a straight-forward approach for analyzing the allowable capacity of pegged mortise and tenon tension connections. The design process included in TFEC 1-10, which is described in more detail in this article, is based on the NDS yield model equations and provides a similar level of performance to steel dowel connections. The TFEC approach is based on physical testing and numerical modeling of connections, coupled with corresponding reliability analyses. This bulletin provides a review and further explanation of the development of the design equations included in the TFEC approach, along with a numerical example of their application.

Introduction

Traditional timber frame construction consists of large timbers commonly connected using mortise and tenon joints. These joints are traditionally secured together with wood dowels (pegs). A tension load on the tenoned member is transferred into the mortised member by the pegs acting in double shear, as shown Figure 1. Construction of engineered timber frame structures, which use pegged mortise and tenon connections, is increasing in popularity for residential and commercial construction. While timber frame construction has a long-standing historical precedence, modern building codes and design considerations have required a more stringent look at these connections. The objective of this paper is to describe the process and the background behind the design method included in the TFEC 1-10.
Brungraber was perhaps the first in recent times to investigate these connections [3]. Schmidt et al conducted significant research at the University of Wyoming, which included substantial physical testing along with a new proposed peg shear yield mode [4,5,6,7]. Further physical testing by others yielded similar results [8,9].

The existing NDS yield model and dowel bearing equations are based on steel-doweled connections. Steel dowels have a substantially higher bearing strength than wooden pegs, so a different set of dowel bearing equations is required. Church and Tew investigated the effects of grain orientation on dowel bearing capacity, although substantial testing was required to account for all of the various parameters [10]. A spring-in-series approach was instead used to combine the various orientations [11].

Bulleit investigated the levels of performance expected from the current NDS yield model equations using steel-dowel connectors [12]. A similar analysis was conducted on pegged connections to ensure that a similar level of performance could be expected when designing both pegged and steel-dowel connections [13].

**Lateral Design Procedure**

The yield limit equations included in the NDS as well as TFEC 1-10 are used to design mortise and tenon connections where the applied load causes the tenon to withdraw from the mortise. Pegs in mortise and tenon connections are loaded in double shear, with the corresponding yield
mode equations shown in Table 1. The NDS Mode IV cannot occur in normal configurations of mortise and tenon joints. This limitation occurs because the Mode IV dowel yield points occur well outside the mortise cheek bounds [4]. Thus, Mode IV does not need to be analyzed in pegged connections. This paper focuses on using the existing yield mode equations included in the NDS coupled with the new peg shear yield mode (Mode V) to determine a design capacity of pegged connections.

The nominal design capacity $Z$ for a single peg in a connection is the minimum of the four yield modes shown in Table 1. The nominal design capacity must be multiplied by all applicable NDS adjustment factors to achieve an allowable design capacity $Z'$. For a connection consisting of multiple pegs, $Z'$ is multiplied by the total number of pegs in the connection to arrive at the total joint design capacity. Use of yield mode equations in Table 1 will result in a similar level of reliability as steel-bolted timber connections [13], provided that the requirements of the TFEC 1-10 are maintained. Several of the more notable requirements of TFEC 1-10 are that:

- The mortise cheeks (side walls) must be at least as thick as the tenon.
- The peg specific gravity must be greater than or equal to the timber specific gravity, and be at least 0.57. The limits on specific gravity are a result of full-scale testing conducted on pegged joints. Due to limits in the test data, the upper bounds of peg specific gravity must not exceed 0.73. When denser pegs are used, a specific gravity of 0.73 may be assumed for analysis purposes.
- The yield mode equations were developed for pegs between 0.75 inches to 1.25 inches in diameter. As rigorous testing and modeling has not been conducted on peg diameters outside of this range, the equations may not be valid for smaller or larger pegs. The use of smaller or larger diameter pegs is not precluded by the TFEC 1-10, although no design guidance is provided.
- Mortise and tenon connections must be loaded such that the tenoned (main) member is loaded by the pegs parallel to the grain. Pegs may not be used to transfer load to a tenon at any other angle to the grain.
Table 1 - Yield Limit Equations for Pegged Double Shear Connections

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Capacity</th>
</tr>
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</table>
| $I_m$      | $Z = \frac{D I_m F_{em}}{R_d}$  

(1) |
| $I_s$      | $Z = \frac{2D I_s F_{es}}{R_d}$  

(2) |
| $III_s$    | $Z = \frac{2k_s D I_s F_{em}}{(2 + R_s) R_d}$  

(3) |
| $V$        | $Z = \frac{\pi D^2 F_{vy}}{2R_d}$  

(4) |
where:

\[ D \] = peg diameter, (in)

\[ F_{em} \] = tenon dowel bearing strength, (psi)

\[ F_{es} \] = mortise dowel bearing strength, (psi)

\[ F_{e\parallel} = 4770G_p^{1.32} \], parallel to grain dowel bearing strength, (psi)

\[ F_{e\perp} = 4900G_p\sqrt{G_T} \], perpendicular to grain dowel bearing strength, (psi)

\[ F_{e\theta} = \frac{F_{e\parallel}F_{e\perp}}{F_{e\parallel}\sin^2\theta + F_{e\perp}\cos^2\theta} \], angle to grain dowel bearing strength. (psi)

\[ G_p \] = specific gravity of the peg material (from NDS Table 11.3.2A [1])

\[ G_T \] = specific gravity of the timber material where \( G_T < G_p \) (from NDS Table 11.3.2A [1])

\[ F_{vy} = 4850G_pG_T^{0.75} \], effective dowel shear strength, (psi)

\[ F_{yb} \] = bending yield strength of the peg, (psi)

\[ = 24850G_p^{1.13} \] (approximated from Wood Handbook [14])

\[ k_3 = \sqrt{-1 + \frac{2(1 + R_e)}{R_e} + \frac{2F_{yb}(2 + R_e)D^2}{3F_{em}l_s^2}} \]

\[ K_\theta \] = correction factor to account for loading at an angle to the grain

\[ = 1 + \frac{\theta}{360} \] (1 ≤ \( K_\theta \) ≤ 1.25)

\[ l_m \] = tenon thickness, (in)

\[ l_s \] = mortise cheek thickness, (in)

\[ R_e \] = main to side member dowel bearing ratio

\[ = \frac{F_{em}}{F_{es}} \]

\[ R_d \] = reduction term to calibrate yield capacity to allowable capacity, where

\[ = 4K_\theta \] (Modes I_m, I_s),

\[ = 3.2K_\theta \] (Mode III_s)

\[ = 3.5 \] (Mode V)

\[ \theta \] = maximum angle of load to any timber grain, (deg) \( 0^\circ \leq \theta \leq 90^\circ \)
**Dowel Bearing Capacity**

Dowel bearing equations included in section 11 of the NDS are based on steel dowels. However, wooden dowels (pegs) are more compliant than steel dowels, resulting in different dowel bearing equations [11]. These dowel bearing equations are listed in the notes section of Table 1, and should always result in a lower dowel bearing capacity than a same-sized steel dowel connection. The yield model’s bounds on peg diameter \((0.75 \text{ in} \leq D \leq 1.25 \text{ in})\) and specific gravity \((G_r < G_p)\) apply to dowel bearing capacity as well.

Peg equations are based on the assumption that they are always being compressed perpendicular to their grain, regardless of grain orientation in the connecting timbers. When the connecting timber is loaded parallel to the grain (typically the tenon), the timber dowel bearing strength greatly exceeds that of the peg (it acts essentially as a rigid interface), leaving a dowel bearing relationship independent of the timber properties. When the connecting timber is loaded perpendicular to the grain (commonly the mortise member), both the connecting timber and peg have dowel bearing strengths in the same order of magnitude, resulting in both materials affecting the joint dowel bearing strength [11].
The following design examples show the yield mode equations and how they are commonly used to analyze a typical pegged mortise and tenon joint subjected to withdrawal loads.

**Design Example # 1**

**Problem Statement**

Determine the withdrawal capacity of the following connection, which consisting of two 1-inch diameter red oak pegs securing a tying joint in a Eastern white pine frame, as shown below in Figure 3. The tendency for withdrawal is caused by wind loading.

![Figure 2 – Exploded view of a mortise and tenon joint](image)

**Given Information**

\[
\begin{align*}
D &= 1 \text{ inch} \\
G_p &= 0.68 \text{ (NDS Table 11.3.2A)} \\
G_T &= 0.36 \text{ (NDS Table 11.3.2A)} \\
l_m &= 2 \text{ inches} \\
l_s &= 3 \text{ inches} \\
\theta &= 90 \text{ degrees} \\
C_o &= 1.6 \text{ for wind (NDS Table 2.3.2)}
\end{align*}
\]
Solution

The main member (tie beam) is loaded parallel to the grain, so the dowel bearing strength is calculated as

\[ F_{em} = F_{e\|} = 4770 (0.68)^{1.32} = 2867.0 \text{ psi} \]

The side member (mortised post) is loaded perpendicular to the grain, so the dowel bearing strength is calculated as

\[ F_{es} = F_{e\perp} = 4900 (0.68)\sqrt{0.36} = 1999.2 \text{ psi} \]

The yield shear strength of the peg is

\[ F_{vy} = 4850 (0.68)(0.36)^{0.75} = 1532.8 \text{ psi} \]

and the yield dowel bending strength of the peg is

\[ F_{yb} = 24850 (0.68)^{1.13} = 16071.7 \text{ psi} \]

In order to calculate the yield mode equations, several constants need to be calculated. They are

\[ R_e = \frac{2867.0}{1999.2} = 1.434 \]

\[ k_i = -1 + \sqrt{\frac{2(1+1.434)}{1.434} + \frac{2(16071.1)(2+1.434)^2}{3(2867.0)(3)^3}} = 1.196 \]

\[ K_\theta = 1 + \frac{90}{360} = 1.25 \]

\[ R_d (I_m, I_s) = 4(1.25) = 5 \]

\[ R_d (III_s) = 3.2(1.25) = 4 \]

From here, the nominal capacity of each mode is calculated

\[ Z_{lm} = \frac{(1)(2)(2867.0)}{5} = 1146.8 \text{ lb} \quad (TFEC 1-10 3.4-1) \]

\[ Z_{lm} = \frac{2(1)(3)(1999.2)}{5} = 2399.0 \text{ lb} \quad (TFEC 1-10 3.4-2) \]

\[ Z_{III_s} = \frac{2(1.196)(1)(3)(2867.0)}{(2 + 1.434)(4)} = 1497.2 \text{ lb} \quad (TFEC 1-10 3.4-3) \]
The nominal design capacity per peg is the minimum yield mode capacity

\[ Z = \min(1146.8, 2399.0, 1497.2, 687.9) = 687.9 \text{ lb} \]

which is limited by a mode V peg shear failure. The design capacity per peg can be adjusted for load duration, such that the capacity of each peg is

\[ Z' = C_p Z = 1.6(687.9) = 1100.7 \text{ lb} \]

The tying connection consists of two pegs, so the maximum allowed withdrawal force that can be resisted by this connection is calculated as

\[ P = n Z' = 2(1100.7) = 2201.4 \text{ lb} \]

**Design Example # 2**

The following design example shows the yield mode equations and how they are commonly used to analyze a typical pegged mortise and tenon joint subjected to withdrawal loads.

**Problem Statement**

Determine the nominal capacity for a 1-inch diameter white oak peg used to connect a knee brace to a post in a Douglas fir frame, as shown below in Figure 3.

![Figure 3 – Exploded view of a mortise and tenon joint](image-url)
Given Information

\[ D = 1 \text{ inch} \]
\[ G_p = 0.73 \text{ (NDS Table 11.3.2A)} \]
\[ G_T = 0.50 \text{ (NDS Table 11.3.2A)} \]
\[ l_m = 1.5 \text{ inches} \]
\[ l_s = \text{ minimum(2 inches, 4 inches)} = 2 \text{ inches} \]
\[ \theta = 45 \text{ degrees} \]

Solution

The main member (tenoned knee brace) is loaded parallel to the grain, so the dowel bearing strength is calculated as

\[ F_{em} = F_{ei} = 4770(0.73)^{1.32} = 3148.5 \text{ psi} \]

The side member (mortised post) is loaded at an angle to the grain, so the dowel bearing strength needs calculated using Hankinson’s formula as

\[ F_{es} = F_{e\theta} \]
\[ F_{e\perp} = 4990(0.73)\sqrt{0.5} = 2529.3 \text{ psi} \]
\[ F_{es} = F_{e\theta} = \frac{(3148.5)(2529.3)}{(3148.5)\sin^2 45 + (2529.3)\cos^2 45} = 2805.2 \text{ psi} \]

The yield shear strength of the peg is

\[ F_{vy} = 4850(0.73)(0.5)^{0.75} = 2105.2 \text{ psi} \]

and the yield dowel bending strength of the peg is

\[ F_{yb} = 24850(0.73)^{1.13} = 17413.3 \text{ psi} \]

In order to calculate the yield mode equations, several constants need to be calculated. They are

\[ R_e = \frac{3148.5}{2805.2} = 1.122 \]
\[ k_1 = -1 + \sqrt{\frac{2(1+1.122)}{1.122} + \frac{2(17413.3)(2+1.122)^2}{3(3148.5)(2)^3}} = 1.581 \]
\[ K_\theta = 1 + \frac{45}{360} = 1.125 \]
From here, the nominal capacity of each mode is calculated

\[
Z_{w} = \frac{(1)(1.5)(3148.5)}{4.5} = 1049.5 \text{ lb} \quad (TFEC \ 1-10 \ 3.4-1)
\]

\[
Z_{\text{III}} = \frac{2(1)(2)(2805.2)}{4.5} = 2493.5 \text{ lb} \quad (TFEC \ 1-10 \ 3.4-2)
\]

\[
Z_{\text{III}} = \frac{2(1.581)(1)(2)(3148.5)}{(2 + 1.122)(3.6)} = 1771.0 \text{ lb} \quad (TFEC \ 1-10 \ 3.4-3)
\]

\[
Z_{v} = \frac{\pi (1)^2 (2105.2)}{2(3.5)} = 944.8 \text{ lb} \quad (TFEC \ 1-10 \ 3.4-4)
\]

The nominal design capacity of the joint is the minimum yield mode capacity

\[
Z = \min(1049.5, \ 2493.5, \ 1771.0, \ 944.8) = 944.8 \text{ lb}
\]

which is limited by a mode V peg shear failure. Therefore, the nominal capacity of the knee brace in tension is 945 lb. Any applicable adjustment factors may be made at this time according to section 3.4.9 of TFEC 1-10. Note that a mode V peg shear design equation is based on empirical- and numerical-testing data, where the yield point was defined using the 5% dowel diameter offset method [15]. This method can often limit joint capacity on the basis of serviceability issues (excessive joint deformation), rather than the basis strength issues (ultimate capacity) [7].

In order to achieve the nominal capacity of the joints in the above examples, the peg locations must be properly detailed to ensure that there is adequate relish, tenon edge distance, and mortise side distance. This process of joint proportioning is outlined in section 3.4.8 of TFEC 1-10 [2] with additional recommendations being provided in the Commentary to section 3.4.8.

**Tabulated Design Values**

Most timber frame connections are fabricated using several predominant species of timber and peg, with standardized tenon thickness. In order to facilitate the design process, the design values for a single peg in a mortise and tenon connection have been tabulated in Table 2. The tabulated values may be used in lieu of the yield limit equations found in Table 1, provided that:
- the tenon thickness is at least 1 ½” thick
- each mortise cheek is at least as thick as the tenon
- the tenoned member is loaded parallel to the grain
- the mortised member is loaded perpendicular to the grain

Table 2 – Tabulated peg capacities for various peg / timber species combinations.

<table>
<thead>
<tr>
<th>Peg Species</th>
<th>Frame Species</th>
<th>3/4&quot; Dia</th>
<th>1&quot; Dia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>G=0.36</td>
<td>G=0.38</td>
</tr>
<tr>
<td>Red Maple</td>
<td>Eastern White Pine</td>
<td>330</td>
<td>264</td>
</tr>
<tr>
<td></td>
<td>Lodgepole Pine</td>
<td>587</td>
<td>611</td>
</tr>
<tr>
<td></td>
<td>Eastern Hemlock</td>
<td>358</td>
<td>373</td>
</tr>
<tr>
<td></td>
<td>Lodgepole Pine</td>
<td>637</td>
<td>664</td>
</tr>
<tr>
<td></td>
<td>Sugar Maple</td>
<td>364</td>
<td>379</td>
</tr>
<tr>
<td></td>
<td>R. Oak</td>
<td>647</td>
<td>672</td>
</tr>
<tr>
<td>White Ash</td>
<td>397</td>
<td>358</td>
<td>373</td>
</tr>
<tr>
<td></td>
<td>637</td>
<td>664</td>
<td>703</td>
</tr>
<tr>
<td>Sugar Maple</td>
<td>401</td>
<td>416</td>
<td>432</td>
</tr>
<tr>
<td>R. Oak</td>
<td>647</td>
<td>672</td>
<td>714</td>
</tr>
<tr>
<td></td>
<td>364</td>
<td>379</td>
<td>401</td>
</tr>
<tr>
<td>Locust</td>
<td>404</td>
<td>421</td>
<td>445</td>
</tr>
<tr>
<td></td>
<td>718</td>
<td>748</td>
<td>792</td>
</tr>
<tr>
<td>W. Oak</td>
<td>415</td>
<td>433</td>
<td>458</td>
</tr>
<tr>
<td></td>
<td>738</td>
<td>769</td>
<td>814</td>
</tr>
</tbody>
</table>

1. Tabulated design values (Z) in lbs, may be multiplied by all applicable adjustment factors per NDS Table 10.3.1.
2. Proper detailing for end, edge, and spacing distances is required to achieve tabulated values. See TFEC 1-10 Section 3.4.8.
3. Tabulated values are based on 1½” thick tenons with 2” thick mortise cheeks except where designated by a *, signifying 2” thick tenons are required.
Conclusion

The design capacity of pegged mortise and tenon joints subjected to withdrawal loads is easily obtained using the method outlined in TFEC 1-10. This method is similar the yield-mode design equations included in the NDS for steel-dowel connectors, and provides a similar level of reliability.

Detailing of pegged mortise and tenon joints to achieve the full design withdrawal capacity is discussed in a subsequent Technical Bulletin produced by the TFEC.

References


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